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DENSITY PROFILE OF A SELF-GRAVITATING POLYTROPIC TURBULENT FLUID IN THE CONTEXT OF MOLECULAR CLOUDS

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Abstract. Molecular clouds (MCs) are the birthplaces of stars in the galaxies. Their complex physics is governed by gravity, supersonic turbulence, magnetic fields and, in the general case, an isothermal equation of state (EOS). We propose a model of a spherically symmetric, self-gravitating and turbulent MC with a compact small core in whose vicinity the EOS changes from isothermal to one of a *hard polytrope*. Within this framework, we obtain of an equation for the density profile function. Under the assumption of steady state, we show that the total energy per unit mass is an invariant with respect to the fluid flow. A method is also proposed to obtain approximate solutions for the density profile in a power-law form. It yields four solutions corresponding to different polytropic exponents and energy balance equations for a fluid element. One of them: a density profile with slope −3 and polytropic exponent $\Gamma = 4/3$, matches with observations and numerical works.

1. INTRODUCTION

Molecular clouds (MCs) are the sites of star formation in galaxies. They are characterized by very low temperatures ($T \sim 10 - 30$ K) and consist mostly of molecular hydrogen well mixed with small amounts of dust (see Ballesteros-Paredes et al. 2020, for a review). Their thermodynamics can be described – in the general case – by isothermal equation of state (EOS): $P \propto \rho^{\Gamma}$, with $\Gamma = 1$. However, the dense parts of MCs obey an EOS of a *hard polytrope* with $\Gamma > 1$ (Federrath & Banerjee 2015; Kritsuk, Norman & Wagner 2011).

The complex physics and evolution of MCs is governed mostly by gravity, supersonic turbulence and magnetic fields. Accretion from the surrounding medium and feedback from new-born stars and supernovae play an essential role at advanced evolutionary stages (Mac Low & Klessen 2004; McKee & Ostriker 2007; Hennebelle & Falgarone 2012; Klessen & Glover 2016).

MCs display fractal structure in a large range of spatial scales $0.001\,\mathrm{pc} \le L \le 100\,\mathrm{pc}$ (Elmegreen 1997; Hennebelle & Falgarone 2012) wherein mean density varies from about 10^2 cm^{-3} at $L \sim 100\,\mathrm{pc}$ up to > 10^5 cm^{-3} at the scales of prestellar cores ($L \le 0.1\,\mathrm{pc}$). An important problem of the theory of star formation is to understand the link between the observed structure of MCs and their physics. There are two general approaches to describe the cloud structure. The first one is to develop methods to extract local substructures and then to derive their physical properties. Some widely used clump-finding techniques are GAUSSCLUMPS (Stutzki & Guesten 1990), CLUMPFIND (Williams et al. 1994), DENDROGRAMS (Rosolowsky et al. 2008) etc. The other general approach is to trace the general cloud structure by analysis of its global characteristics, e.g. probability distribution function (PDF) of mass density.

The contribution of this study is to present a theoretical model of the density profile in the vicinity of the core of a self-gravitating spherically symmetric turbulent cloud at later stages of its evolution. It can explain the shape of the PDF in its high-density parts in a good agreement with simulations and observations.

2. MASS-DENSITY PDF AND THE PHYSICS BEHIND

Mass-density PDF traces the general structure of MCs in terms of abstract scales. Let us denote the PDF with $P(s)$, where $s = \ln(\rho / \rho_c)$ is the log-density and ρ_c is a normalization unit. Then one can define an abstract scale, associated with the density level s , through the equation:

$$L = L_c \left(\int_s^{s_0} P(s') ds' \right)^{1/3} \quad , \tag{1}$$

where L_c is a normalizing quantity, usually the total cloud size. One can distinguish two basic types of PDFs, which are explained by different physics. A lognormal shape (a Gaussian of log-density) is attributed to domination of supersonic isothermal turbulence over gravity, thermal pressure and magnetic fields. On the other hand, the emergence of a power-law tail (PLT) is often interpreted as a manifestation of dominance of gravity over turbulence and other forces. The PDF type is determined by the cloud evolution. As shown from numerical simulations, supersonic isothermal turbulence dominates at early stages after the cloud formation and then the PDF is lognormal (Vazquez-Semadeni 1994; Passot & Vazquez-Semadeni 1998; Kritsuk et al. 2007; Federrath et al. 2010). Observations of regions of low/no star-forming activity also display lognormal (column-density) PDFs (Kainulainen et al. 2009; Lombardi et al. 2014; Schneider et al. 2015a,b, 2016). Supersonic turbulence creates dilutions and condensations (clumps) in the medium. Gravity takes over in dense clumps and a PLT emerges in the corresponding density range, while the diluted material is still characterized by a lognormal PDF (Klessen 2000; Dib & Burkert 2005; Slyz et al. 2005; Kritsuk, Norman & Wagner 2011; Veltchev et al. 2019). For theoretical explanations of the PLT and its evolution we refer the reader to the works of Girichidis et al. (2014), Donkov & Stefanov (2018,2019), Li (2018), Jaupart & Chabrier (2020).

At the final stage of MC evolution a *second* PLT forms (Fig.1). Numerical (Kritsuk, Norman & Wagner 2011) and observational studies (Schneider et al. 2015c) provide evidence of this phenomenon whereas the physics behind is still poorly understood. Several hypotheses have been suggested: rotation of contracting cloud core(s), strong magnetic fields in the densest parts of the cloud, change in the thermodynamics – transition from isothermal state at larger scales to polytropic one at small scales (Kritsuk, Norman & Wagner 2011; Schneider et al. 2015c). All those factors possibly act together. In this report we suggest a possible explanation for the second PLT with a change in the EOS from isothermal at larger scales to one of a *hard polytrope* at small scales. This model is a first step toward reproduction of the PDF in dense cloud cores and therefore some idealizations are permitted (see next Section).

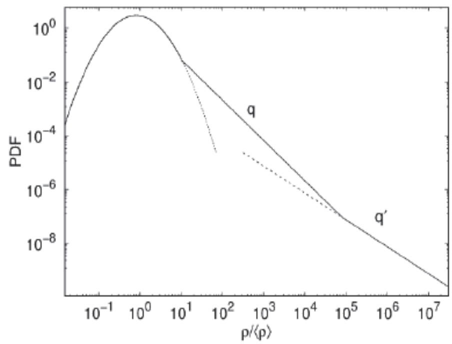

Figure 1: Example of mass-density PDF with a lognormal part and two PLTs, with slopes q and q'.

3. OUR MODEL OF AN EVOLVED MOLECULAR CLOUD

The cloud model was originally presented in Donkov & Stefanov (2018) and developed further in Donkov & Stefanov (2019). Here we recall its basic assumptions and modify its thermodynamics.

The cloud is modeled as gaseous, spherically symmetric, and self-gravitating ball. The matter accretes through its outer boundary of radius L_c and passes through all spatial scales down to a very small and dense core. The core is homogeneous with radius $L_0 = L_c$, density ρ_0 and mass M_0 which increases slowly in respect to the accretion timescale. Hereafter we consider scales L and densities ρ to be normalized to the cloud scale L_c and density ρ_c at the cloud boundary, correspondingly.

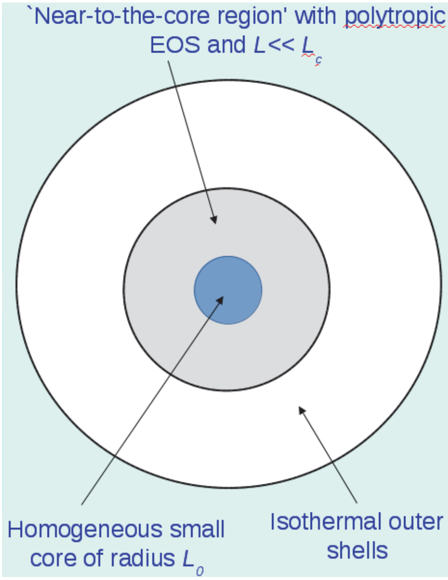

Figure 2: Schematic representation of our cloud model.

The dimensionless characteristics of the cloud core are: $L_0 = 1$ and ρ_0 ? 1. Hence the spatial scales and the densities outside the core span the ranges: $L_0 \leq L \leq 1$ and $\rho_0 \leq \rho \leq 1$, correspondingly.

We assume a profile $\rho(L)$ of mass density which is related to the volume-weighted PDF $P(s)$ through the equation:

$$P(s)\,ds = -3L^2 dL \qquad , \qquad s = \ln(\rho(L)) . \tag{2}$$

The cloud is turbulent. The turbulence is locally homogeneous and isotropic in each cloud shell with radius L and volume $dV = 4\pi L^2 dL$. All spatial scales in the range $L_0 \leq L \leq 1$ belong to the inertial range of the turbulent cascade and therefore the dissipation can be neglected in the equations. Each scale in the cloud is in a steady state regarding both the macro-states (motions of the fluid elements) and the micro-states (thermal motion of the molecules). This conception is similar to the so called “simple bath-tube model” of Burkert (2017).

In this paper we modify the cloud thermodynamics. The gas is considered isothermal in the outer shells (Ferriere 2001), far from the central high-density region, while it obeys the EOS of a *hard polytrope* near to the core, where the gravitational potential of the latter becomes dominant in the energy balance.

4. EQUATIONS OF THE MEDIUM AND EQUATION FOR THE DENSITY PROFILE

We consider the equations of the medium near to the cloud core as follows:

- the Euler equation for a fluid element (Navier-Stokes equation without a dissipative term)

$$\frac{\partial}{\partial t}\vec{u} + \vec{u}\cdot\nabla\vec{u} = -\frac{1}{\rho}\nabla P_{gas} - \nabla\varphi \qquad (3)$$

- the continuity equation

$$\frac{\partial}{\partial t}\rho + \nabla\cdot\left(\rho\vec{u}\right) = 0 \qquad (4)$$

- the Poisson equation for the gravitational potential φ

$$\Delta\varphi = 4\pi G\rho \qquad (5)$$

- and the polytropic EOS of the gas with power index $\Gamma > 1$ (P_0 is a normalizing quantity for the gas pressure)

$$P_{gas} = P_0\rho^{\Gamma}\,. \qquad (6)$$

We implement the equation of polytropic state (6) in the Eq. (3) and then multiply the obtained equation with an infinitesimal displacement $d\vec{r} = \vec{u}dt$ in direction of the vector field $\vec{u}$. In the obtained scalar product we introduce the dimensionless variables $v^2 = u^2 / c_s^{\,2}$ and $\phi = \varphi / c_s^{\,2}$, where c_s is the isothermal sound speed in the outer shells of the cloud. Then we make use of the assumption for steady state and neglect partial derivatives in respect to the time. Finally one gets the following equation:

$$d\left[\frac{1}{2}v^2 + \frac{\Gamma}{\Gamma-1}\rho^{\Gamma-1} + \phi\right] = 0\,. \qquad (7)$$

This is an equation for energy conservation per unit mass of the fluid element during its motion through the cloud scales near to the core. In fact this is the Bernoulli equation for our model. Taking into account the spherical symmetry of the model, one can replace the total differential in (7) with full derivative d / dL, because the averaged in time (in respect to the locally homogeneous and isotropic turbulence) motion of the fluid element is radial:

$$\frac{d}{dL}\left[\frac{1}{2}v^2 + \frac{\Gamma}{\Gamma-1}\rho^{\Gamma-1} + \phi\right] = 0\,. \qquad (8)$$

A more detailed derivation of equations (7) and (8) for isothermal EOS can be found in Donkov & Stefanov (2018). The considered case of EOS of a *hard polytrope* will be developed in a forthcoming paper (Donkov et al. 2020, in preparation).

Now we present the explicit form of the terms in equation (8). We apply the assumptions of our model as well the assumption that the solution for the density profile is in a power-law form: $\rho(L) = L^{-p}$, with exponent in the range $1 < p \leq 3$. The kinetic energy term consists of the turbulent and accretion kinetic energy per unit mass and reads:

$$v^2 = {v_t}^2 + {v_a}^2 = T_0 L^{2\beta} + A_0 \rho^{-2} L^{-4} = T_0 L^{2\beta} + A_0 L^{2p-4}, \qquad (9)$$

where the turbulent term scales according to Larson's law (Larson 1981): $v_t \propto L^{\beta}$, and the exponent takes values in the range: $0 \leq \beta \leq 1$. The explicit form for the accretion energy (the second term on the right hand side) is obtained from the continuity equation (4) written in spherical coordinates and making use of the steady-state assumption. Coefficients T_0 and A_0 are the ratios of the turbulent kinetic energy and accretion kinetic energy per unit mass at the cloud boundary to the thermal kinetic energy (${c_s}^2$ - for isothermal EOS) per unit mass, respectively. The second term in equation (8) is the thermal energy per unit mass for polytropic EOS. Assuming a density profile in a power-law form, it reads:

$$\frac{\Gamma}{\Gamma - 1} \rho^{\Gamma - 1} = \frac{\Gamma}{\Gamma - 1} L^{-p(\Gamma - 1)}. \qquad (10)$$

The last term in (8) is the gravitational potential of the system. It consists of three subterms: gravity of the shells below the position of the fluid element, gravity of the shells above the fluid element, and gravity of the core. Its explicit form is:

$$\phi = -3G_0 \frac{L^{2-p}}{3-p} \left[1 - \left(\frac{L_0}{L} \right)^{3-p} \right] - 3G_0 \frac{1 - L^{2-p}}{2-p} - G_1 \frac{1}{L}.$$

(11)

We consider the motion of the fluid element near to the core – hence $L : L_0$ and the first term on the r. h. s. of equation (11) vanishes (it accounts for the gravity of the shells below the fluid element). So equation (11) is simplified to:

$$\phi \approx -3G_0 \frac{1 - L^{2-p}}{2-p} - G_1 \frac{1}{L}. \qquad (12)$$

The physical meaning of the coefficients G_0 and G_1 is simple. They are, correspondingly, the ratios of the gravitational energy of the cloud shells (outside the core) per unit mass and of the gravitational energy of the core per unit mass, both measured at the cloud boundary, to the thermal (for isothermal EOS) energy per unit mass ${c_s}^2$.

Finally one can write the equation (8) in an explicit form:

$$A_0 L^{2p-4} + T_0 L^{2\beta} + 2\frac{\Gamma}{\Gamma-1} L^{-p(\Gamma-1)} - 3G_0 \frac{1-L^{2-p}}{2-p} - G_1 \frac{1}{L} = \mathrm{const}\left(L\right). \qquad (13)$$

This is an equation for the density profile $\rho\left(L\right) = L^{-p}$.

5. POSSIBLE SOLUTIONS OF THE EQUATION

There is no exact solution of Eq. (13), for any value of p . Our aim is to find an approximate solution using only the terms of leading order. Such solution can be obtained when the leading-order terms are at least two, have equal exponents and balance each-other (have opposite signs). To obtain the leading-order solution for the index p we compare in pairs the exponents of the different terms in Eq. (13) and obtain a root. The next step is to assess the exponents of all terms in the equation using the obtained value of p. Here one has to take into account that the leading terms are those with the smallest exponent since $L < 1$. There are four possible solutions for *hard polytropes*. They are briefly presented below.

Let us assume that the gravity of the core is balanced by the accretion kinetic term and the thermal term. Then, the exponents of these terms must be equal. The exponent of the accretion kinetic term is $2p-4$ while that of the term which accounts for the gravitational energy of the core is -1 . From $2p-4=-1$, one gets the root $p=3/2$. The gravity-of-the-core term and the thermal term are of the same order only if $\Gamma=5/3$. This value for the polytropic index is obtained from $-p(\Gamma-1)=-1$. The energy balance provided by the leading order terms of Eq. (13) when $p=3/2$ and $\Gamma=5/3$ is described by

$$A_0 + 2\frac{\Gamma}{\Gamma-1} - G_1 = 0. \qquad (14)$$

The other three solutions are obtained in a similar manner. When $p=3/2$ and $1<\Gamma<5/3$ only the accretion kinetic term provides the balance against the gravity of the core. The energy balance in this case is

$$A_0 - G_1 = 0. \qquad (15)$$

The thermal term provides the energy balance against the core gravity when $p=2$ and $\Gamma=3/2$. This root results in the following energy balance

$$2\frac{\Gamma}{\Gamma-1} - G_1 = 0. \qquad (16)$$

The last possible approximate solution corresponds to $p=3$ and $\Gamma=4/3$. In this case the thermal term provides the energy balance against both the gravity of the core and the ram pressure of the infalling outer shells. This energy balance is expressed by the equation

$$2\frac{\Gamma}{\Gamma-1} - G_0 - G_1 = 0. \qquad (17)$$

6. CONCLUSIONS

In this report we propose a model of self-gravitating turbulent molecular cloud. Some common features with previous models are the spherical symmetry and the assumption that the physics of the cloud is governed by gravity, turbulence and accretion (Larson 1969, Shu 1977; Hunter 1977; Donkov & Stefanov 2019). Unlike the first three of the referred models, there is no explicit dependence on time, i.e. the molecular cloud is considered to be in a quasi-steady state. Another major difference between the current model and the previous ones is that the equation of state of the molecular gas is polytropic with $\Gamma > 1$ instead of isothermal.

With these assumptions and using the equations of the medium in case of equation of state of a *hard polytrope* we obtain an equation of conservation of the total energy per unit mass. Under the assumption of mass-density profile in a power-law, the latter equation results in an equation for the exponent of this profile p .

We obtain four approximate solutions for p and constraints for the polytropic index Γ . One of the most significant results in this report is that the sole change in thermodynamics (from isothermal to *hardly polytropic* equation of state) can explain the observed density profile $\rho \propto L^{-3}$ at high densities.

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